Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
Vol. 11, 2013, ISSN 1584-4536

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Series A

Functional Equations

Approximation and Convexity

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of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 3–17.

A note on invariant subspaces and the solution of some classical functional equations

Jose María Almira (Jaén) Khader Faiez Abu-Helaiel (Jaén)

Abstract. We study the continuous solutions of several classical functional equations by using the properties of the spaces of continuous functions which are invariant under some elementary linear trans-formations. Concretely, we use that the sets of continuous solutions of certain equations are closed vector subspaces of $C(\mathbf{C}^d, \mathbf{C})$ which are invariant under affine transformations $T_{a,b}(f)(z) = f(az+b)$, or closed vector subspaces of $C(\mathbf{R}^d, \mathbf{R})$ which are translation and dilation invariant. These spaces have been recently classified by Sternfeld and Weit, and Pinkus, respectively, so that we use this information to give a direct characterization of the continuous solutions of the corresponding functional equations.

1 Motivation

We study the continuous solutions of several classical functional equations by using the properties of the spaces of continuous functions which are

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Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 19–28.

Some remarks on the generalized mean upper/lower semideviations of order p from a target

Alina-Ramona Baias
(Cluj-Napoca)

Abstract. In this paper we extend the classical mean semivariance from a target, to a more general risk function, *the generalized mean semideviation of order p from a target*. We state and prove its main properties and we give dual representations for both the lower and the upper semideviations. Moreover, the structure of the portfolio optimization problem, having those functions as objective is studied.

Key Words: convex risk functions, conjugate functions, dual representation, semideviations of order p

MSC 2000: 90C25, 91B30, 49N15

1 Background in notations and definitions

Let $(\Omega, \mathfrak{F}, \mathbb{P})$, be an *atomless probability space* where Ω is the support space, $\mathfrak{F}$ is a $\sigma-$algebra on Ω and $\mathbb{P}$ is the probability measure. For a

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Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 29–39.

The convergence of Henig efficient solutions net for vector equilibrium problems

Adela Elisabeta Capătă
(Cluj-Napoca)

Abstract. The purpose of this paper is to present a result which assures the convergence of a net of Henig efficient solutions of a strong vector equilibrium problem to a solution of the problem.

1 Introduction

Considering Z a real topological linear space, $C \subseteq Z$ a nontrivial pointed convex cone, A a nonempty subset of a topological space E, B a nonempty set, and $\varphi : A \times B \to Z$ a given bifunction, the strong vector equilibrium problem is formulated as:

(VEP) find $\bar{a} \in A$ such that $\varphi(\bar{a}, b) \notin -C \setminus \{0\}$ for all $b \in B$.

Many existence results of solutions of vector equilibrium problems are given in the hypothesis of a cone with nonempty interior (see, for instance [1, 2, 3, 7, 12]). But, there are important ordered topological linear spaces

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Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 41–51.

Some applications of Hopf's identity on divided differences

Ioan Gavrea (Cluj-Napoca) Mircea Ivan (Cluj-Napoca) Rozica Moga (Cluj-Napoca)

Abstract. We give some mean-value theorems involving divided differences, which extend recent results of this type.

Key Words: Divided difference, mean-value theorems, convexity
MSC 2000: 26A24, 41A05, 26A51

1 Introduction and preliminary results

We use simple techniques to extend the mean-value formulas in [4, 9, 8, 10, 11], to enlarge the class of functions for which they are satisfied, and to provide simple proofs of the existing ones.

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Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 53–57.

Uniform approximation of functions by Bernstein-type operators

Adrian Holhoş
(Cluj-Napoca)

Abstract. For the class of bounded functions defined on $[0,1]$ and continuous on $(0,1)$ we give a characterization of the functions which can be uniformly approximated by Bernstein-type operators.

Key Words: positive linear operators
MSC 2000: 41A36

1 Introduction

Bernstein polynomials were introduced by S. N. Bernstein [1] in 1912 to constructively solve the problem of K. Weierstrass [12] of uniformly approximating the continuous functions by using polynomials. The Bernstein operators are defined by

$$B_n(f,x)=\sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k}f\left(\frac{k}{n}\right),\quad x\in[0,1],\ n\geq 1.$$

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Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 59–85.

A bivariate infimal convolution formula and the maximal monotonicity of the parallel sum

Szilárd László
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Abstract. By making use of some well known infimal convolution formulas, we give several regularity conditions, both closedness and interior point type, that ensure the maximal monotonicity of the parallel sum of two maximal monotone operators of Gossez type (D) in arbitrary Banach spaces. We also obtain similar results for the parallel sum of the Gossez monotone closure of two maximal monotone operators.

Key Words: infimal convolution, monotone operator, maximal monotone operator of Gossez type (D), representative function.
MSC 2000: 47H05, 46N10, 42A50

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❍ This work was supported by a grant of the Romanian Ministry of Education, CNCS-UEFISCDI, project number PN-II-RU-PD-2012-3-0166.

Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 87–94.

Semi-infinite optimization problems and their first order approximations

Dorel I. Duca (Cluj-Napoca) Augusta Raţiu (Cluj-Napoca)

Abstract. In this paper, we study the relationships between the set of optimal solutions of a semi-infinite optimization problem and its approximated optimization problem.

Key Words: Semi-infinite optimization, invex function, approximated optimization problem.
MSC 2000: AMS subject classifications: 90C46, 90C59, 90C34.

1 Introduction

We consider the optimization problem in the following form:

$$(\mathrm{P}) \qquad \begin{array}{ll} \min & f(x) \\ \text{subject to} & x \in X \\ & g_t(x) \leq 0,\ t \in T, \end{array}$$

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Series B

Mathematical Interdisciplinary Research

Annals of the Tiberiu Popoviciu Seminar
of Functional Equations, Approximation and Convexity
ISSN 1584-4536, vol 11, 2013, pp. 97–111.

Mathematical model for the skin penetration by nanoparticles carried drugs

Gabriela Nuţ
(Cluj-Napoca)

Abstract. In this paper we use the multigrid method to model the skin penetration of a drug transported by nanoparticles. The purpose of this paper was to determine the concentration of the substance at a given depth in the skin, where it can not be practically measured, and to calculate the concentration that has to be applied on the surface so that, at a given layer of the skin, the substance would have an appropriate concentration. The skin is modeled as a multilayer having different diffusion coefficients. The concentration at a given depth is computed using different types of conditions on the borders of the domain and between the different layers.

Key Words: multigrid method, diffusion process, time dependent differential equation

MSC 2000: 65Z99

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