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Series A

Functional Equations

Approximation and Convexity

On solutions of $f(x) + f(a_1x) + \cdots + f(a_Nx) = 0$ and related equations

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ABSTRACT. We study some properties of the solutions of the functional equation

$$f(x) + f(a_1x) + \cdots + f(a_Nx) = 0,$$

which was introduced in the literature by Mora, Cherruault and Ziadi in 1999, for the case $a_k = k + 1$, $k = 1, 2, \dots, N$ [3]. For our study we use an idea recently introduced by Popa and Rasa [7] in connection with Fréchet functional equation.

1 Motivation

The functional equation

$$(1.1) \quad f(x) + f(a_1x) + \cdots + f(a_Nx) = 0,$$

was introduced in the literature by Mora, Cherruault and Ziadi in 1999, for the case $a_k = k + 1$, $k = 1, 2, \dots, N - 1$ [3] and studied by Mora [4] and

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Old-New Methods for Computing Subdifferential Formulae for Convex Risk Functions

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ABSTRACT. This paper presents a new approach for computing subdifferential formulae, which is an alternative for the irregular risk functions that can not be described by utility functions. The alternative is not entirely new, in fact is based on old techniques and results from convex analysis. The idea is to combine the already known methods of convex optimization with the modern risk measures (which can be irregular) in order to provide better tools to quantify the risk. The only necessary requirement is the convexity of the risk function.

KEY WORDS: convex risk functions, convex subdifferential, mean absolute deviation

MSC 2000: 90C25, 91B30, 49N15

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An Existence Result for Henig Globally Efficient Solutions of Vector Equilibrium Problems

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ABSTRACT. The purpose of this paper is to present an existence result for Henig globally efficient points of the strong vector equilibrium problem under compactness, upper semicontinuity and weak concavity assumptions.

1 Introduction

Many existence results of solutions of vector equilibrium problems are given in the hypothesis of a cone with nonempty interior (see, for instance [1, 2, 6, 12]). But, there are important ordered topological linear spaces whose ordering cones have an empty interior. For example, when $Z := L^p(T, \mu)$, where (T, μ) is a σ -finite measure space and $p \in [1, +\infty[$, the cone

$$C := \{u \in L^p(T, \mu) \mid u(t) \geq 0 \text{ a.e. in } [0, T]\}$$

has an empty interior.

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Sequential Optimality Conditions for Equilibrium Problems

LIANA CIOBAN
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ABSTRACT. In this paper we give necessary and sufficient sequential optimality conditions in order to characterize the solutions of the general equilibrium problem but also for some particular cases of it, in case when no regularity conditions are fulfilled. Among the special instances of our results, we rediscover also the sequential optimality conditions given for variational inequalities.

KEY WORDS: equilibrium problems, perturbation theory, sequential optimality conditions

MSC 2000: 90C25, 47A55, 90C46, 90C33

1 Introduction

In what follows we consider the general (Stampacchia type) equilibrium problem introduced in [12] which consists in finding an element $\bar{x} \in X$ such that

$$(PEP) \quad F(\bar{x}, x) + \Phi(x, 0) \geq \Phi(\bar{x}, 0) \quad \forall x \in X,$$

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Restricted Directional Approximations of Some Generalized Convexity Properties

MIHAIL GĂIANU
(TIMIȘOARA)

ABSTRACT. In this paper we study the restricted convexity with respect to a given set of directions and a given set of representing points from the space. The relation with the classical convexity and some kinds of generalized convexities is considered from the approximation point of view. The process of convergence of sequences of restricted convexities is considered, when the set of directions and the set of representing points indefinitely refine.

KEY WORDS: approximation, convexity, convexity with respect to a set, restricted directional convexity with respect to a set

MSC 2000: 52A05, 52A30

1 Introduction and basic concepts

Restricting various mathematical concepts and procedures to some favorite directions or orientations in the space is frequently useful in pattern

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Vector Variational-like Inequalities and Set-valued Optimization Problems

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ABSTRACT. In this paper, we present several kinds of generalized invexity for set-valued mappings and some relationships between a set-valued optimization problem and vector variational-like inequalities are established.

KEY WORDS: Generalized invexity · Set-valued optimization problem · Vector variational-like inequalities

1 Introduction

In 1998, Giannessi [14] first used, so called, Minty type vector variational inequality (in short, *MVVI*) to establish the necessary and sufficient conditions for a point to be an efficient solution of a vector optimization problem (in short, (*VOP*)) for differentiable and convex functions. Since then, several researchers have studied (*VOP*) by using different kinds of *MVVI* under different assumptions, see [1, 2, 4, 13, 19, 21] and the references therein. Consequently, vector variational inequalities have been

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A smooth variational description involving Dini-Hadamard-like decoupled subgradients

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ABSTRACT. It seems that the original motivation to write such an article came from the intention to obtain a smooth variational description for Dini-Hadamard-like subgradients with a key role in deriving further exact calculus rules for this kind of objects. Our analysis relies on a decoupled version of Dini-Hadamard-like subgradients and involves various concepts such as the notion of spongiously decoupled local minimizer and the notion of Dini-Hadamard-like differentiability.

KEY WORDS: directional derivatives, Dini-Hadamard and Dini-Hadamard-like subdifferentials, decoupled constructions, smooth variational description of subgradients, Dini-Hadamard-like differentiability, sponge, spongiously local minimizer, calm functions

MSC 2000: 49J52, 49J53, 90C56

1 Introduction

Nonsmooth phenomena have been well known for a long time in mathematics and applied sciences, too. To deal with nonsmooth functions, sets

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Connections Between Vector Optimization Problems, Their Solutions and Saddle Points

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ABSTRACT. Considering a vector optimization problem, we attach to it the $(0, 1) - \eta$ approximated vector optimization problem. We study the connections between the efficient solutions and saddle point of these two problems.

KEY WORDS: vector optimization problem, $(0, 1) - \eta$ approximated vector optimization problem, efficient solution, saddle point.

MSC 2000: 90C26, 90C30, 90C46

1 Introduction and preliminaries

To a vector optimization problem one can attach different vector duals or approximated vector optimization problems and then study the connections between their solutions and saddle points (see, for example [1]). Here we

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About a Lexicographic Multicriteria Bottleneck Problem

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ABSTRACT. The aim of the present paper is two-fold. The first is to study the structure of the set of optimal solutions of the lexicographic multicriteria bottleneck optimization problem. The second is to study the structure of the set of all optimal solutions with pipeline property for this type of problem.

KEY WORDS: lexicographic multicriteria bottleneck problem, bottleneck function, minimum point with pipeline property, convex set

MSC 2000: 90C10, 90C27, 90C29

1 Introduction

In this paper we present the lexicographic optimization problems with p objective functions of bottleneck type (or time type), generic denoted by us with (LpBP). As useful papers regarding the lexicographic bottleneck problems we recall [2], [8], [9]. In [2] the authors study combinatorial optimization problems with bottleneck objective function, where any feasible

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On Uniform Polynomial Stability for Evolution Operators on the Half-line

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ABSTRACT. The aim of this paper is to give necessary and sufficient conditions for uniform polynomial stability of evolution operators with uniform polynomial growth. Well-known results for uniform exponential stability are extended to the case of uniform polynomial stability.

KEY WORDS: evolution operator, uniform polynomial stability

MSC 2000: 34D05, 34E05

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Series B

Mathematical Interdisciplinary Research

A Fixed Point Approach to First Order Differential Equations and Systems

OCTAVIA NICA
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ABSTRACT. The aim of the present paper is to present a less used operator approach that we call Burton's method to first order differential equations and systems. For systems, this method is combined with an other less known technique, namely the method of matrices convergent to zero. We will present some existence results using Perov's and Schauder's fixed point theorems.

KEY WORDS: Fixed point, Cauchy problem, Matrix convergent to zero, Schauder's fixed point theorem, Perov's fixed point theorem.

MSC 2000: 34A12, 47J25

1 Introduction

It is well-known that the method of successive approximations resulted from Banach's Contraction Principle was applied for the study of existence, uniqueness and approximation of the solution for Fredholm and Volterra integral equations in [9]. In this book, it was presented the calculation of

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Mathematical Understanding of the Autologous Stem Cell Transplantation

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ABSTRACT. A simple mathematical model is provided for understanding the cell dynamics after autologous stem cell transplantation, concluding about the effectiveness of this therapeutic procedure for acute myeloid leukemia and suggesting some biological and clinical directions of further possible investigation.

KEY WORDS: mathematical model, medical application, dynamic system, numerical simulation, stem cell transplantation, acute myeloid leukemia

MSC 2000: 92C50, 37N25, 37M05, 34A34

1 Introduction

1.1 Medical background

Acute myeloid leukemia (AML) is a cancer of the hematopoietic bone marrow stem cells, due to the expansion of an abnormal mutated stem cell clone which leads to the inhibition of surrounding normal cells. Current

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Operatorial Tau Method for Some Delay Equations

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ABSTRACT. The paper extends the applicability of our Matlab package *Cheb-pack* to find Chebyshev polynomial approximations of the solutions of some delay differential equations. The results are accompanied by an error analysis and the numerical examples confirm the theory and prove the importance for practice of this approach.

KEY WORDS: Delay equations, Chebyshev polynomials, tau method

MSC 2000: 34K28, 65J10, 65Q05

1 Introduction

There is a growing interest in the numerical solution of functional differential equations, due to their importance in modeling of environmental, biological and physical processes, where they provide a more realistic simulation. The numerical solving of such type of equations is complicated by the

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Modeling and Solving The Per-unit Royalty Case in a Differentiated Stackelberg Duopoly

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ABSTRACT. The aim of this article is to study a particular multilevel programming problem, generated by a concrete economic problem related to technology transfer area: two firms compete on the market in a differentiated Stackelberg model and there is a technology transfer between the two firms based on a per-unit royalty licensing contract. A mathematical model is attached to this particular economic problem and an optimal solution is found. The model consists in a multilevel parametric programming problem, in which both the upper and the lower level functions are to be maximized under some given conditions.

KEY WORDS: multilevel programming, parametric programming problem, per-unit royalty, differentiated Stackelberg model

MSC 2000: 90B50, 90C29, 90C31

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